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Parametrisation of Momentum Transport due to Cumulus-Convection

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Abstract

In this study a parametrisation of the vertical transport of horizontal momentum by cumulus convection for use with the Kühlungsborn Mechanistic Climate Model (KMCM) is presented. The KMCM uses a relaxation scheme in the style of Frierson [8]. Existing schemes are commonly based on mass-flux convection schemes. In order to apply these existing schemes to the KMCM the cloud-scale horizontal momentum equations is redesigned and a formulation for the cloud-scale vertical mass-flux is presented.
The parametrisation has been tested in 'offline' simulations with the KMCM and it is found to be in good agreement with the results presented in previous studies. An analysis of the effects of the implemented cloud-scale pressure gradient by Zhang and Cho [27] on the cloud-scale horizontal momentum is made. It is found to be of small influence on the CMT. This confirms recent findings by Romps [18] and Richter and Rasch [17].

Zusammenfassung

In dieser Arbeit wird eine Parametrisierung der Effekte des vertikalen Transports von horizontalem Impuls durch Cumulus Konvektion (CMT) präsentiert. Die Parametrisierung wird für die Verwendung im Kühlungsborn Mechanistic Climate Model (KMCM) modelliert. Im KMCM wird ein Relaxations-Ansatz in der Form von Frierson [8] verwendet, um Konvektion zu modellieren. Bisherige Parametrisierungen des CMT benutzen Kovektions-Routinen, die auf Massenfluss Konvektionsparametrisierungen basieren. Frühere CMT Parametriserungen werden überarbeitet um im KMCM angewendet werden zu können. Zu diesem Zweck wird die horizontale Wolken-Impuls Gleichung überarbeitet und eine Formulierung für den vertikalen Massen-Fluss in der horizontalen Größenordnung der Wolken hergeleitet.
Die Parametrisierung wurde mittels 'offline' Rechnungen des KMCM getestet. Die Ergebnisse liefern gute Übereinstimmung mit in vorherigen Studien veröffentlichten Ergebnissen. Eine Analyse der implementierten Parametrisierung des Druck-Gradienten zwischen Wolke und Umgebung von Zhang and Cho [27], zeigt das dieser geringen Einfluss auf den errechneten Impulstransport hat. Dies bestätigt Ergebnisse aus Veröffentlichungen von Romps [18] und Richter and Rasch [17].

To my son Cajun, who is great inspiration in being completely oblivious of it.
To Lara without her loving care this would not have been possible.
To all those guiding me on way.

Contents

Chapter 1.

Introduction

The mechanics of the atmosphere and climate have become of increasing interest in recent times. Even before the pressing of imminent climate change, daily weather forecasts have made people want to understand the physics behind weather. The Primitive Equations physically describe momentum tendencies and temperature tendencies within a fluid. The atmosphere though is not a fluid in the general sense. It is a composite of different gasses. For the general dynamics water vapour and general air are the determining factors, but as earth is subject to constant perturbations it is prevented from reaching its equilibrium state. This is due to radiative exchange with outer space. The effects of these daily perturbations result in ever reoccurring change in the temperature distribution and interactions of ionized layers with the earth's magnetic fields. These effects improve the complexity of the earth's climate and make it more complicated to understand than can be grasped by the primitive equations.
Tools devised to help with forecasting the evolution of the climate and have been much improved in recent years are models of the atmosphere. Diverse in kind they can be used to model the dynamics of a region or of the general circulation, providing a playground for researchers to test and prove theories and find out about mechanics of the atmosphere.
The model has its own limitations. The most crucial one lies with the sheer computational resources needed to model the atmosphere, and the limitations of numerics in solving differential equations. A set of partial differential equations which has to be solved in four dimensions is something that may not be solved for every molecule in the atmosphere. To address this limitation the atmosphere is divided into columns, each the area of a fraction of the earth's surface. These are divided into layers. Each of these volumes is considered to be homogeneous in all relevant physical properties. Certain boundary conditions are devised to describe the limits of the model and solve

the equations.
These models are based on the general idea that each flow consists of two components, its mean component and the deviations from it. Averaging over an area will remove the deviations from the mean flows and leave the means to compute with. The equations are however non-linear and not all of the effects by deviations are lost, certain turbulent fluxes remain and connect the small scale deviations with the large scale flows. Since in models these small scales are outside the resolution, expressions for these turbulent fluxes are needed in order to correctly model the climate. This process of finding influential small scale phenomena and expressing them through large scale quantities is called parametrisation. Parametrisations are used wherever the small scales have important impact on the large scales.
These parametrisations do not only have a large impact on models, but also express an important aspect of understanding the processes defining the dynamics of the atmosphere.

This work is about the parametrization of a specific problem; the momentum transport by convection associated to cumulus clouds or deep convection, Cumulus Momentum Transport (CMT). Several studies have been published on CMT, the earliest proposed by Ooyama [16]. Subsequently the parametrisation by Schneider and Lindzen [19] is the one mostly accepted and improved in subsequent studies. Using this parameterization scheme Helfand [11] found an enhancement of the winter Hadley circulation and the wind fields in his model were found to be closer to experimental observations. Zhang and McFarlane [25] incorporated an improved parametrisation, which includes cloud-scale pressure-gradients, derived by Zhang and Cho [27] (ZC95) into the Canadian Climate Centre (CCC) GCM. They as well observed an enhanced Hadley circulation in summer and an improvement in their wind field, similar to that of Helfand [11]. Gregory, Kershaw, and Inness [10] achieved similar results when implementing their improved version of the ZC95 CMT parametrisation into the Hadley Centre Climate Model.
Wu, Liang, and Zhang [21] undertook a 20-yr simulation, in which ZC95 was implemented, into the Community Research Model developed by the National Center for Atmospheric Research in order to evaluate the effect of the CMT on climate simulation utilising long-term climate statistics. A secondary meridional circulation, characterized by strong upward motion along the strongest ascending belt of the Hadley circulation and a downward motion north and south of the belt, within the Hadley circulation was found. In a study by Wu et al. [23] a modification of the simulated tropical circulation,

precipitation, cloud distribution and radiation due to CMT was found. Similar results were achieved by Cheng and Xu [5]. In more recent years secondary effects of CMT are studied. A study by Zhou and Kang [29] for example found that CMT is important to determine the meridional scale of the tropical waveguide. It can lead to wave instabilities as well as induce upscale momentum and energy transfer from the planetary-wave scale to the large scale.

Parametrisations are always specific to their certain model and its surroundings. The subject of this study is to design a parametrisation of the CMT to be used with the Kühlungsborn Mechanistic Climate Model (KMCM) [2] to model earth's climate. The KMCM uses a parametrisation for cumulus convection into which the parametrisation presented here is fitted. Therefore the principles of convection, stability of the atmosphere and the concept of this convection parametrization will be laid out in the second chapter. The parametrisation of CMT shall follow in the third chapter, building on what has been found before. The fourth and fifth chapters will centre on the implementation and evaluation of the parametrization. The evaluation will be performed using 'offline' calculations of the KMCM with the parametrisation included. In conclusion an overview of the work will be given followed by a discussion of the limitations of the parametrisation presented here and suggestions for future improvements.

Chapter 2.

Convection

Convection is buoyant forcing manifested in our environment. Air rises because it is positively buoyant and vice versa. As simple as this may be if condensed down to its essence, convection is a complex phenomenon that has led to many years of research over the past decades. It is not simply one thing rising form one place in the atmosphere to another, but also what it carries with it while doing so.

Convection is a prominent effect throughout the atmosphere. It changes the vertical structure of the atmosphere substantially. In higher latitudes convection usually affects a few tens of meters. In this case the parametrisation chosen is a diffusion of physical properties, which is well understood. Close to the equator though the instabilities reach up to 200 hPa which results in large scale cumulus convection over several hundreds of meters. Due to this concentrated convection the vertical wind speeds become substantially larger than in the rest of the atmosphere, where vertical wind speeds are considered negligible in relation to the large scales. These winds result in mass transport, which in return carries humidity, heat, momentum and tracers. The identification of these processes resulted in the necessity of an adequate parametrisation of the effects of cumulus convection. Atmospheric convection releases latent heat through condensation and absorbs it through evaporation. It transports heat as well as moisture and, the subject of this work, it transports momentum.

2.1. Characterisation of stratification

In order to physically describe convection the picture of finite parcels of air is introduced. Convection is driven through buoyant forcing, which is described by the Archimedes law:

$$F = g \cdot \delta V(\rho_s - \rho_p)$$

The force on a submerged body of volume δV and of density ρ_p is equal to the difference of its density and the density of the fluid ρ_s it is submerged in, times the gravity constant. In atmospheric physics, forces and energies are represented as forcing or energy per mass, accordingly the buoyant forcing can be represented as:

$$F = g \cdot \frac{\rho_s - \rho_p}{\rho_p}$$

Here the law for fluids is transferred to a gas. The properties of gasses are assumed to develop according to the ideal gas law. Further, to describe convective processes, parcels are assumed to be lifted without exchanges with the environment, i.e. the process is assumed to be adiabatic. The ideal gas law is used to reformulate, to achieve $F(T_p, T_s)$:

$$F = g \cdot \frac{T_p - T_s}{T_s}$$

Therefore, the parcel is accelerated when the temperature of the parcel T_p at a given height is higher than the temperature of the surrounding air T_s at that height. To determine whether or not a given parcel will be accelerated upward, resulting in convection, the temperature dependency $T_p(z)$ with height needs to be determined. During ascent the atmosphere does work on the parcel, but in assuming the process to be adiabatic no energy is added to it, i.e $Q = 0$. The heating rate Q is defined as:

$$Q = C_v \frac{dT}{dt} + p \frac{dV}{dt} = 0$$

Combining this relation with the ideal gas law results in:

$$\frac{1}{\gamma - 1}\frac{1}{T} + \frac{1}{V}\frac{dV}{dt} = 0 \tag{2.1}$$

$$\gamma = \frac{C_p}{C_v} \geq 1$$

We now assume γ to be independent of temperature and integrate eq. 2.1, yielding:

$$TV^{\gamma-1} = p^{(1-\gamma)/\gamma}T = pV^{\gamma} = \textit{const.}$$

To find the temperature gradient for an adiabatic ascent we differentiate with respect to height, z and get:

$$\frac{dT}{dz} + \frac{1-\gamma}{\gamma}\frac{T}{p}\frac{dp}{dz} = 0$$

$\frac{dp}{dz}$ represents the pressure change experienced by the parcel. The characteristic of an adiabatic ascent requires that the pressure of the parcel is always that of its surroundings, implying: $\frac{dp}{dz} = \frac{dp_s}{dz}$. In hydrostatic equilibrium the vertical pressure gradient has to be balanced by gravity i.e. $\frac{dp}{dz} = -\rho_s * g$. Joining these findings we get:

$$\frac{dT}{dz} = -\frac{\gamma - 1}{\gamma}\frac{g}{R}\frac{T}{T_s}$$

The dry adiabatic lapse rate Γ_d defines the atmospheric temperature profile for which a parcel that ascends or descends adiabatically without condensation of water vapour is always at the temperature of its surroundings $T = T_s$. This way it is ensured that the process undergone by the parcel is adiabatic:

$$\frac{dT}{dz} = \frac{dT_s}{dz} = -\frac{\gamma - 1}{\gamma}\frac{g}{R} = -\Gamma = -\frac{g}{c_p}$$

The stability criteria may now be reformulated based upon the temperature gradients. The parcel temperature T_p may only be larger than that of the surroundings if the adiabatic lapse rate is larger than the temperature gradient of the atmosphere, hence:

$$\textit{stable:}\ \frac{dT_s}{dz} < -\Gamma$$

$$\textit{unstable:}\ \frac{dT_s}{dz} \leq -\Gamma$$

So far it was assumed that the parcel undergoing scrutiny is moved without water vapour condensing or water evaporating. Examining the effect of the freezing of water is beyond the scope of this study, and thus only water vapour will be examined. It will further be assumed that condensate is immediately precipitated. The concentration of water vapour in air is expressed as vapour pressure e. The absolute humidity expressed as the number density n_v of water molecules or the mass density, ρ_v follows:

$$n_v = \frac{e}{kT}$$
$$\rho_v = \frac{e}{R_v * T}$$

Here R_v is the gas constant for water vapour. Consequently a vapour pressure is defined for every specific temperature, at which the vapour is in equilibrium with its condensed phase. At the equilibrium point the exchanges between the two phases are equal; evaporation and condensation are in equilibrium. When the temperature drops below this point vapour will be condensed, so that equilibrium is reinstated; when it rises above water will be evaporated, again until equilibrium is reached. In general within the atmosphere the water vapour is seldom in equilibrium with its condensed phase. Often the vapour pressure e of gas is below the equilibrium vapour pressure e_s. Therefore the approximation which we made above is relatively accurate. The equilibrium vapour pressure is a function of temperature, whereby at a specific temperature the vapour will condense or evaporate until it reaches the corresponding equilibrium vapour pressure. Consequently, there may be cases in which no liquid phase is evident within the parcel, therefore no equilibrium with a condensed phase may be reinstated. A specific temperature T_s is defined, which is the exact point at which a condensed phase would start forming if the parcel were to be cooled further. Until this specific point, as seen above, the temperature will change with the dry adiabatic lapse rate Γ_d.

As the parcel rises it will reach the equilibrium vapour pressure temperature T_s. Subsequently its vapour will begin to change its phase as the parcel is cooled further by rising, in order to achieve equilibrium with its condensed phase. Condensing vapour

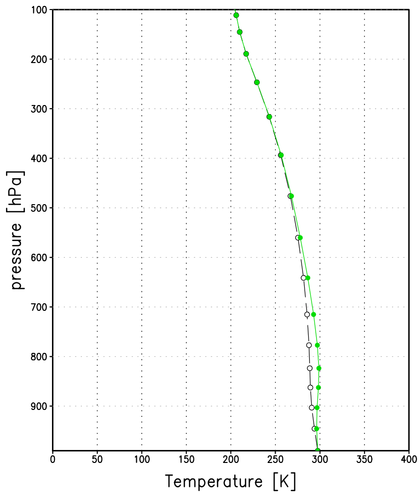


Figure 2.1: green,solid: T_r
black,dashed: T.
Example profiles from the convection routine of the KMCM. Note how the profiles align at around $350hPa$, this is the cap for convective processes. Beyond this point density becomes so low that the approximations made for the moist and dry adiabatic lapse rates no longer apply.

results in the release of latent heat following:

$$q = \frac{\rho_v}{\rho}\frac{dT}{dz} = \frac{\mathcal{L}}{c_p} * \frac{dq_e}{dz} \tag{2.2}$$

Here q, the specific humidity, is used. Since the specific humidity is kept at the equilibrium specific humidity when rising, the change in equilibrium humidity gives the change in humidity, i.e. condensed vapour. $\mathcal{L}$ is the latent heat per unit mass.

A correction is added to the dry adiabatic lapse rate to describe the temperature evolution beyond the equilibrium point temperature T_e:

$$\Gamma = \begin{cases} \Gamma_d = \frac{g}{c_p} & \textit{if } T < T_e \\ \Gamma_m = \frac{g}{c_p} - \frac{\mathcal{L}}{c_p}\left|\frac{dq_e}{dz}\right| & \textit{if } T \geq T_e \end{cases}$$

This gives the full definition of temperature changes during adiabatic ascent of an air parcel. The definition of stable stratification of the air column can be derived :

$$\textit{stable: } \Gamma_m \geq \frac{dT_s}{dz}$$
$$\textit{conditionally unstable: } \Gamma_d \geq \frac{dT_s}{dz} \geq \Gamma_m$$
$$\textit{unstable: } \frac{dT_s}{dz} \geq \Gamma_d$$

Another measure for stability is the Brunt-Väisälä-Frequency N^2, it is the frequency with which a parcel would oscillate, were it slightly displaced from it's resting position. The derivation is similar to that of an harmonic oscillator, friction will be neglected.

The force acting on the displaced parcel is the buoyancy force found by Archimedes to be proportional to the difference in densities of displacing ρ_p and displaced ρ_s fluid, the volume displaced by the displacing parcel V and the gravitational constant g.

$$F = -gV \cdot (\rho_p - \rho_s) = M\frac{d^2z}{dt^2} = \rho_p \cdot V\frac{d^2z}{dt^2} \tag{2.3}$$

We defined these effects to be adiabatic, therefore the pressure p of the displaced parcel is always that of it's surroundings. In good approximation the ideal gas equation may be used to describe the physical processes in the atmosphere.

$$p = \rho_s t_s R = \rho_p T_p R \tag{2.4}$$

combining 2.3 and 2.4 gives

$$\frac{d^2}{dt^2}z = g\Big\{\frac{T_p(z) - T_s(z)}{T_s(z)}\Big\} \tag{2.5}$$

Using our findings from above we may formulate a function for the parcel temperature for the dry adiabatic ascent and the moist adiabatic ascent.

$$\begin{aligned}
T_{moist}(z) &= T_p(z_0) + \Big(\frac{g}{c_p} + \frac{\mathcal{L}}{c_p}\frac{dq_{sat}}{dz}\Big)(z - z_0) \\
T_{dry}(z) &= T_s(z_0) - \frac{dT}{dz}(z - z_0) \\
\Rightarrow N^2_{moist} &= \frac{g}{T}\Big(\frac{g}{c_p} + \frac{dT}{dz} + \frac{\mathcal{L}}{c_p}\frac{dq_{sat}}{dz}\Big) \\
N^2_{dry} &= \frac{g}{T}\Big(\frac{g}{c_p} + \frac{dT}{dz}\Big)
\end{aligned} \tag{2.6}$$

Where the latter is for the dry adiabatic case. Following the stability conditions from above, we get:

$$\begin{aligned}
\textit{stable:}\ & N^2_{moist} > 0 \\
\textit{conditionally stable:}\ & N^2_{dry} > 0 > N^2_{moist} \\
\textit{unstable:}\ & 0 > N^2_{dry}
\end{aligned}$$

This concept gives rise to three distinct levels in a certain parcels rise. The level at which water vapour begins to condense is the lifting condensation level (LCL), from where on its ascent will be moist adiabatic. The level at which the atmosphere becomes

Figure 2.2: green,solid: T_r
black,dashed: T
Example profiles from the convection routine of the KMCM. For this example the capping as well as the implicit condition for convection have been deactivated. The level of no buoyancy (LNB)has been marked red.

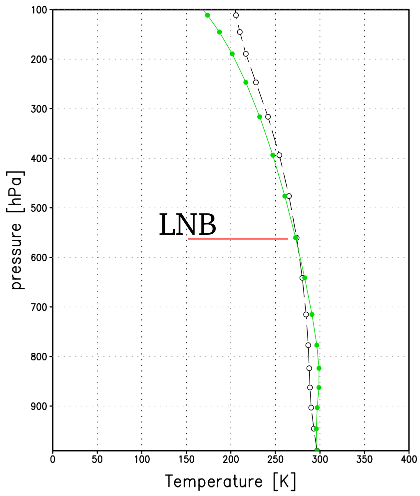
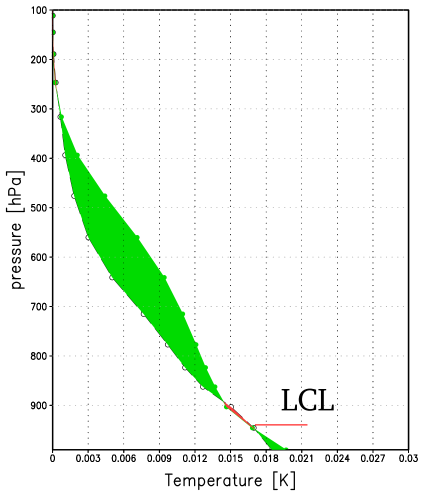


Figure 2.3: green,solid: q_r
black,dashed: q
Example profiles from the convection routine of the KMCM. Note that where ever the moisture is less than the reference moisture the temperature profile follows the dry adiabatic lapse rate. The intersection between the two profiles is the lifting condensation level (LCL).



unstable with respect to the parcel is the level of free convection (LFC). From there on the parcel will remain positively buoyant until it reaches the level of neutral buoyancy (LNB).

2.2. Parametrisation of convection

In the past many parametrisations have been devised aiming to correctly depict convection. There are two broad types of convection schemes, the Mass-Flux-Type-schemes and the Relaxation-Type-Schemes. An Mass-Flux-scheme hinges on mass conservation and continuity equations. Whenever mass is brought into or out of a convective column it carries its respective properties. Though these parametrisations of convection strive towards a holistic description, they have become very complicated over time. A lot of tuning parameters and correction towards not only representability of cloud-scale processes but also the desired outcome takes place.
The latter scheme takes a more practical approach. Convection occurs whenever the atmosphere is in an unstable state and it continues until the convective column is in a state of stability. The Relaxation-Type schemes use this. The state of stability is defined and the process of convection is described as incremental steps towards stability.

The first scheme of the kind was proposed by Betts and Miller [4] and it was further simplified by Frierson [8]. This scheme was developed for maximum simplicity to be applied in General Circulation Models (GCM's). The intricacy and correctness of the parametrisation schemes based on Mass-Flux-schemes come at the price of computation. The advantages of the simplified are that it includes convection, and also has a low requirement on computational resources. The KMCM uses this advantage.

2.2.1. The Frierson convection scheme

The scheme devised by Frierson relaxes humidity and temperature to reference values:

$$\frac{\partial T}{\partial t} = -\frac{T - T_{ref}}{\tau} \tag{2.7}$$

$$\frac{\partial q}{\partial t} = -\frac{q - q_{ref}}{\tau} \tag{2.8}$$

With τ being the relaxation time, a parameter of the scheme. His reference temperature T_{ref} is the virtual pseudoadiabat, taking into account water vapour content of the parcel and the environment. Condensate is approximated to immediately fall out. The humidity reference profile is set to be a fixed humidity distribution relative to the reference temperature profile, following Eq. 2.2 and findings by Neelin and Yu [15].

Three cases of convection are identified depending on the precipitation due to drying P_q and on the precipitation due to warming P_T:

$$P_q = -\int_{p_0}^{p_{LNB}} \delta q \frac{dp}{g}$$
$$P_T = -\int_{p_0}^{p_{LNB}} \frac{c_p}{\mathcal{L}} \delta T \frac{dp}{g}$$

The integrals range from the surface level to the level of no buoyancy (LNB). In the case that the precipitation due to warming P_T is positive this means $CAPE + CINE > 0$ (see Ch. 3.3). $P_q > 0$ implies an over saturated column that is more moisture is in the column than in the reference profile.

Deep Convection

In the first case both, P_T and P_q, are larger than zero; the condition for deep convection is met. To conserve enthalpy the reference temperature profile specified before is corrected:

$$\Delta k = \frac{1}{\Delta p} \int_{p_0}^{p_{LNB}} -(c_p T + \mathcal{L} q - c_p T_{ref} - \mathcal{L} q_{ref}) dp \tag{2.9}$$
$$T_{ref2} = T_{ref} - \frac{\Delta k}{c_p}$$

This done following Betts [3]. In changing the profile by a uniform amount with height, the changes in temperature (in enthalpy units) are opposite to the changes in humidity (in enthalpy units). Consequently it is ensured that $P_T = P_q$. A consequence of this correction is a cooling in the lower troposphere, which is associated with the effects of cooling downdrafts at lower levels.
Precipitation in this scheme is solely dependent on the moisture relaxation, the temperature profile is adjusted to match the moisture relaxation.

Shallow Convection

In case of P_q being negative after the enthalpy conservation done previously (eq. 2.9), their scheme switches to shallow convection. Shallow convection describes those convective processes during which precipitation is evaporated on its way downward. In this case moisture is conserved. To ensure this an additional equation is introduced to the set:

$$0 = \int_{p_0}^{p_{shall}} \Big(-\frac{q - q_{ref}}{\tau_{SBM}} \Big)$$

Chapter 3.

Theory of Parametrisation

In this chapter the parametrisation of the CMT will be derived. The general state of research on the matter shall be presented, along with an analysis and solutions to the problems which were found to apply a cumulus momentum transport parametrisation to the KMCM.

When air is transported upward, due to the instabilities and buoyant processes within the cloud, mass conservation demands air to be entrained i.e. imported into the cloud or convective column. The air transported upward is described as a mass-flux. When it changes new mass is added to the flux or taken away from it, thus resulting in the entrainment or detrainment of air into or out of the convective column, respectively. An exchange of physical properties between environment and convective column occurs wherever air is entrained or detrained.

A convective column (or cumulus cloud) can be pictured to initially move at the same speed as its surroundings. The large vertical velocities associated with convection result in a cloud-scale vertical mass-flux. The changes in the flux, following mass conservation, result in entrainment and detrainment of air into and out of the cloud. These two effects, the vertical transport and the exchange with the large-scale flow, result in changes in the vertical profile of horizontal velocities within the cloud. Over time it starts to deviate from the initial profile of horizontal velocities of the large scale flux surrounding it. The convective column, now moving at parts slower, at parts faster than its surroundings, consequently imposes a drag or friction on the large scale flow. Effectively this results in vertical transport of horizontal momentum from lower levels of the atmosphere to upper levels of the atmosphere. In order to describe these processes a definition of cloud-scale averages is introduced.

3.1. Subscale processes

The set of equations used to physically describe the atmosphere requires the solution of a system of non-linear partial differential equations for each of the components of the atmosphere. Therefore, when regarding the atmosphere, processes are represented as areal or temporal means of these processes and deviations from these means. This is done in order to depict the atmosphere in models. In doing so however only an approximate solution of these equations is achieved. The process of horizontal averaging intersects the earth's atmosphere in volumes, each the horizontal area of a few hundred kilometres squared and a height of a few hundred meters in case of the KMCM. The average of a quantity I over a certain area A according to the Reynolds-Average is:

$$I = \overline{I} + I' \tag{3.1}$$

$$\overline{I} = \frac{1}{A}\int I dA \tag{3.2}$$

$$\overline{\overline{I}} = \overline{I} \tag{3.3}$$

$$\overline{I'} = 0 \tag{3.4}$$

If averaged, the mean part of the quantity shall remain the same (eq. 3.3), while the deviations shall level out so as to amount to nought (eq. 3.4). As the equations are however of non-linear nature not all of these processes disappear. In the case two quantities I and J appear in a product, their average is:

$$\begin{aligned} J &= \overline{J} + J' \\ I &= \overline{I} + I' \\ \overline{I \cdot J} &= \overline{(\overline{J} + J') \cdot (\overline{I} + I')} \\ &= \overline{\overline{I}\,\overline{J}} + \overline{\overline{I} J'} + \overline{I'\overline{J}} + \overline{I' J'} \\ &= \overline{I}\,\overline{J} + \overline{I' J'} \end{aligned}$$

The results are averaged products of the deviations. These processes are understood to be small scale processes influencing the large scale averages, such as CMT for example or wave processes such as gravity waves. To account for these processes they have to be identified from observations or thorough consideration and then need to be parametrised. A parametrisation strives to express these small scale processes through

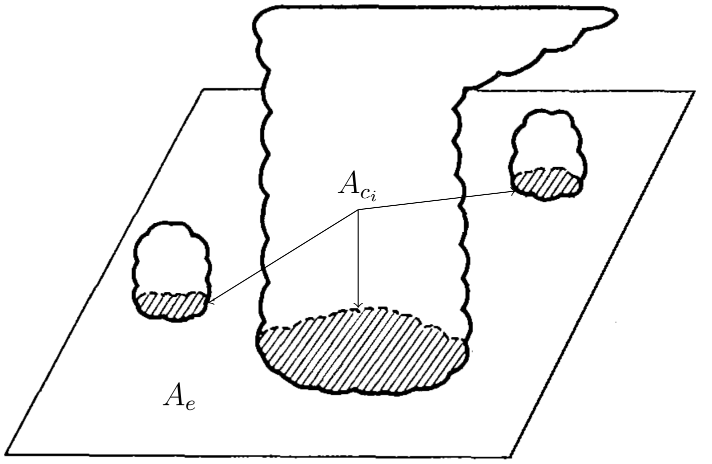


Figure 3.1.: Example area A with environmental area A_e and cloud areas A_{c_i}

the large scale averages, to express the correct physical behaviour of the system. This work will be concerned with cumulus clouds, or more precisely convective columns. The horizontal extend of an individual convective column (i) is denoted as A_{ci}. The combined area covered by convective processes is denoted as A_c. The areas not subject to convection shall be called environment or surroundings of the cloud. For all of these averages are formulated using Eq. 3.2:

$$\overline{I} = \sigma_c \overline{I}_c + (1 - \sigma_c)\overline{I}^e \qquad (3.5)$$
$$\overline{I}^{ci} = \frac{1}{A_{ci}} \int I dA_{ci}$$
$$\overline{I}^{c} = \sum_i \frac{A_{ci}}{A_c} \overline{I}^{ci}$$

Eq. 3.5 addresses the fact that the combined, weighted averages of both, environment and convective column shall give the large scale average. Above $\sigma_c = \sum_i A_{ci}/A = A_c/A$ is defined to be the fractional area subjected to convection. The vertical extend of these convective columns, also referred to as clouds within the text, here is assumed to reach from the level of free convection up to the level of no buoyancy.
In this context a common approximation shall be introduced, the scale separation principle introduced by Arakawa and Schubert [1]. The fractional coverage of convection is assumed to be much smaller than that of the environment and hence the influence of

in-cloud quantities on the large-scale average are assumed to be small i.e.

$$\sigma_c << 1$$
$$\Rightarrow \overline{I} \approx \overline{I}^e \tag{3.6}$$

In the following $\overline{\boldsymbol{I}}^c$ is simplified to $\boldsymbol{I}_c$, denoting the weighted average of cloud quantities.

3.2. The Lindzen and Schneider parametrisation

Most of the modern parametrisations for the vertical eddy momentum flux divergence $\frac{1}{\rho}\frac{\partial}{\partial z}\rho\overline{w'\boldsymbol{u}'}$ by clouds are build upon the paper by Schneider and Lindzen [19] (LS76). They abandoned the idea of $\boldsymbol{u}_c$ being conserved during CMT, in the sense of cloud heat as proposed by Arakawa and Schubert [1] or vorticity as proposed by Fraedrich [7].

3.2.1. Derivation of the LS76 scheme

In the mean Navier-Stokes-Equations the subscale vertical momentum transports are represented by the subscale advection of horizontal momentum $\boldsymbol{u}'$ by vertical momentum w':

$$\frac{1}{\rho}\partial_z\rho\overline{\boldsymbol{u}'w'} \tag{3.7}$$

Schneider and Lindzen derived the relation:

$$\rho\overline{\boldsymbol{u}'w'} = -M_c(\overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}_c)$$
$$\boldsymbol{R}_c = -\frac{1}{\rho}\partial_z(M_c(\overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^c)) \tag{3.8}$$

Where ρ is the density, M_c is the cloud-mass-flux, and $\boldsymbol{u}$,$\boldsymbol{u}_c$ are the large scale average horizontal wind velocities and cloud scale average horizontal velocities, respectively. The vector $\boldsymbol{R}_c$ represents the net friction on the environment due to Cumulus Momentum Transport. It is no friction in the physically correct sense, as friction is a dissipative effect. The original expression by LS76 may therefore be misleading

In order to derive eq. 3.8 they use the separation of cloud and environmental quantities eq.3.5:

$$\rho\overline{\boldsymbol{u}w} = \rho \cdot (\sigma_c \overline{\boldsymbol{u}w}^c + (1-\sigma_c)\overline{\boldsymbol{u}w}^e) \tag{3.9}$$

$$= \rho \sum_i \left(\frac{A_{ci}}{A} \overline{\boldsymbol{u}}^{ci} \cdot \overline{w}^{ci} \right) + (1-\sigma_c)\rho\overline{\boldsymbol{u}w}^e \tag{3.10}$$

using the scale separation principle (eq. 3.6), they show:

$$\rho\overline{\boldsymbol{u}w} \approxeq \rho\overline{\boldsymbol{u}} \cdot \overline{w} + \underbrace{\rho \sum_i \left(\frac{A_{ci}}{A} \overline{\boldsymbol{u}}^{ci} \cdot \overline{w}^{ci} \right)}_{M_c \boldsymbol{u}_c} - \overline{\boldsymbol{u}} \underbrace{\rho \sum_i \left(\frac{A_{ci}}{A} \overline{w}_{ci} \right)}_{M_c = \rho\overline{w}_c} \tag{3.11}$$

$$\rho\overline{\boldsymbol{u}w} = \rho\overline{\boldsymbol{u}} \cdot \overline{w} - M_c(\overline{u} - \overline{\boldsymbol{u}}_c) \tag{3.12}$$

Inserting equations 3.1, 3.3 and 3.4 into equation 3.12, they arrive at 3.8.

The parametrisation by LS76 differs from earlier parametrisations by Ooyama [16] and Arakawa and Schubert [1] through the inclusion of a term proportional to $\frac{\partial \boldsymbol{u}_c}{\partial z}$. However if cloud-scale momentum is assumed conservative in equation 3.8 it becomes equivalent to the previous formulations.

The most substantial approximation leading to the parametrisation is that the fractional area covered by active clouds (convective columns) is much smaller than unity. Only following this approximation cloud and environment may be treated separately, with only certain cloud properties contributing significantly to the areal averages. This approximation is often used in literature up to day. A recent paper by Yano [24], suggests there may be a contradiction in assuming both the fractional area covered by active clouds *and* the relative time-scales of large-scale atmospheric motion τ_L and cumulus convection τ_c to be close to zero i.e. $\sigma_c \rightarrow 0$ *and* $\frac{\tau_C}{\tau_L} \rightarrow 0$. This is of large interest to models of cumulus convection, since both are used in combination during their derivation. In this case only the former approximation is used therefore its use is unproblematic.

The task of incorporating this parametrisation focuses mainly on two components. The vertical cloud mass flux M_c and the horizontal cloud momentum $\boldsymbol{u}_c$. Later papers on the subject focus on the derivation of the cloud-scale momentum equation and on the computation of the cloud-scale mass flux. The latter is for most if not all of the studies undertaken intrinsically given due to the cumulus parametrisation scheme, as CMT schemes have been only applied to mass-flux convection parametrisations. The former is subject to great consideration. Since there is little experimental data on the dynamics within a cloud the derivation of a cloud-scale momentum equation is a complex task. The equations used in this work will be derived in the following sections.

3.3. The cloud-scale mass-flux equation

Two approaches on a expression for the vertical mass-flux are made. One of the approaches will prove to be giving dissatisfying results. None the less it's physical picture is of interest, since the view taken on the processes will be crucial on the form our parametrisation takes and greatly supports the process of understanding the processes inside a convective column.

3.3.1. The distinct-parcel approach

To describe the cloud-scale mass-flux energy considerations of convective processes are used. The convective available potential energy (CAPE) of a parcel (i) is defined by it's buoyancy at each point of the column up to the level where it's buoyancy (see eq. 2.5) is zero (LNB):

$$
\begin{aligned}
CAPE &= \int_{z_i}^{z_{LNB}} a_p dz \\
CAPE &= \int_{z_i}^{z_{LNB}} g\Big\{\frac{T_p(z) - T_s(z)}{T_s(z)}\Big\}dz
\end{aligned}
\tag{3.13}
$$

When regarding a common pre-storm vertical temperature profile T_s and the reference temperature T_p is plotted for comparison, see figure 3.2, areas of positive and negative buoyancy can be identified. The levels of negative buoyant forcing function as a potential barrier preventing the convective process. The work that needs to be done by a parcel rising through that area to its LFC inhibits the convective process, and is hence called convective inhibitive energy The potential energy (CINE):

$$
CINE = \int_{z_i}^{z_{LFC}} a_p dz \tag{3.14}
$$

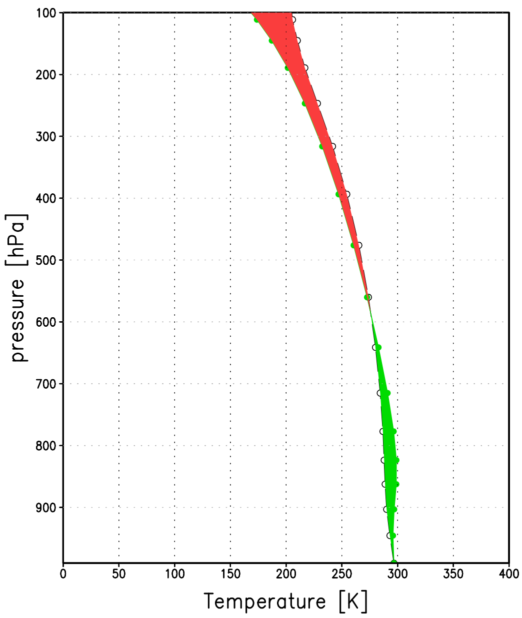


Figure 3.2: Example Temperature profiles from the convection routine of the KMCM.
green: Areas of positive buoyancy $\propto$ CAPE
red: Areas of negative buoyancy $\propto$ CINE
The intersection of the two temperature profiles marks the LNB, the green area is proportional to the CAPE. Before convection an area of negative buoyancy would be apparent below the green area. The levels beyond the LNB are usually disregarded since they are of no importance to the convective process (except being upper boundary).

Here z_{LFC} is the level of free convection. Once the CINE is reduced to zero by atmospheric processes such as advection moisture, heat or through radiative processes, the convection occurs. CINE may also be overcome by the upward momentum of the considered parcel:

$$w_{min} = \sqrt{CINE}$$

Similarly the vertically averaged vertical cloud-scale velocity during convection may be estimated:

$$\overline{w}^z = \sqrt{CAPE}$$

Note that both CAPE and CINE are usually defined as energy per mass quantities, a definition different from the usually used energy definitions, but practical in atmospheric sciences, since density changes vastly with height.

During convection this energy is reduced towards zero as the vertical temperature and moisture profiles approach equilibrium. According to Fritsch and Chapell [9] the temporal evolution of CAPE may be parametrised using the convective time-scale τ_c:

$$\frac{\partial CAPE}{\partial t} = -\frac{CAPE}{\tau_c} \approx \int_{z_i}^{z_{LNB}} \frac{g}{T_s} \frac{\partial T_s}{\partial t}^c$$

Where it is acknowledged that convection changes the vertical temperature profile *only* within the convective column. Using 2.7 and the definition of heating rates the parametrisation by Frierson is reformulated:

$$\frac{\partial CAPE}{\partial t} \approx \int_{z_i}^{z_{LNB}} \frac{g}{T_s} \frac{Q_c}{c_p} \tag{3.15}$$

Which physical picture is conveyed through this parametrisation? Here each parcel is assigned it's respective CAPE, this energy will be transformed into kinetic energy during the process of convection and eventually be dissipated as the parcel overshoots the point of no buoyancy and rises, by excess momentum, into the area of negative buoyancy. Such processes are described by as a damped oscillator, oscillating with the Brunt-Väisälä-Frequency (Eq.2.6). This process is undergone by each of the parcels, each rising along its' respective pseudo adiabats to their respective points of zero buoyancy.
This representation of convection is somewhat complicated to use, since in a physically correct sense, the effects of slower upward moving parcels on faster moving parcels and the overall changing temperature profile of the convective column make it hard to deduce the actual mass-flux at a given height. This becomes even more troubling since the mass-flux must somehow develop over time.
Furthermore using the concept of CAPE only a estimation of the average vertical velocities can be given. To make use of the parametrisation by LS76 however, vertical gradients within the convective column have to be computed.
A different approach to this problem is needed.

3.3.2. The level approach

Some consideration of the applied convective parametrisation has to be done. Information is only gained on the heating rate at which each respective level of our convective column changes it's temperature in each time step τ_c.
Using this information it can be deduced how much work is done by this level, resulting

in cooling or in case the convection results in heating, how much work is done on it. The heating rate, which is gotten from the convective scheme following Frierson [8], therefore has to be balanced by the rate of work done in form of vertical cloud-scale mass transport. Therefore, if the column is viewed as a whole, a redistribution of thermal energy through the mass-flux is taking place. Take notice that this parametrisation is not in fact used to compute this redistribution of thermal energy, the opposite is done. Here the effective vertical redistribution of thermal energy is used to compute the accompanying mass-flux.

It is assumed here that each level of the convective column is a isolated system during convection i.e. that it's internal energy is supposed to be constant. A change in internal energy through heating (see 2.2.1) consequently has to be balanced through a change in the work done:

$$\begin{aligned} dU &= \delta Q + \delta W \\ \frac{\partial U}{\partial t} &= 0 = \frac{\partial Q}{\partial t} + \frac{\partial W}{\partial t} \\ Q_c(z) &= -w(z)a(z) \;\Rightarrow\; M_c = \frac{Q_c\rho}{a(z)} \end{aligned} \tag{3.16}$$

Physically speaking, the picture applied here is not the representation through parcels rising upwards. Effects of vertical gradients of the mass-fluxes are in this formulation intrinsically given. The 'reverse engineering' through the heating rates implies this. An expression is found to express the vertical mass-flux on behalf of the convective heating rate. However an expression for the acceleration in 3.16 is not as clearly to be found. The acceleration of an air parcel can be expressed through buoyant forcing following equation 3.17:

$$a(z) = g\left\{\frac{T_p(z) - T_s(z)}{T_s(z)}\right\} = g \cdot (\frac{T_p}{T_s} - 1) \tag{3.17}$$

It is not clear whether this expression fits the framework applied here. Physically speaking the change in kinetic energy is accompanied by an acceleration. From this it is expected to have a vertical momentum equation which needs solving. The form of this equation would be analogous to the horizontal momentum equations i.e.:

$$\frac{\rho\partial w_c}{\partial t} + \rho\nabla(\boldsymbol{u}_c w_c) = -\partial_z p_c + ... \tag{3.18}$$

It is seems only consequent to then use eq. 3.16 to express $\frac{\partial w_c}{\partial t}$ using the heating rate specified by the convection scheme:

$$-\rho Q_c(z) w_c(z) + \rho \nabla(\boldsymbol{u}_c w_c) = -\partial_z p_c + ... \tag{3.19}$$

A boundary condition for the solution of the equation cloud-scale vertical momentum equation 3.19 would be $w_c(z_0) = \overline{w}$. An increase of the vertical velocities is expected over the duration of convection as well as interactions with the environment through advection. However the task of correctly specifying this equation is left for future studies. For the remainder of this work it is assumed that equation 3.19 is sufficient. The acceleration, following eq. 3.17, is assumed to be g. This is justified as a parametrisation, which is found to yield sufficient results. Thus the equation for the cloud-scale mass-flux used is:

$$M_c = \frac{Q_c \rho}{g} \tag{3.20}$$

3.4. The cloud-scale horizontal momentum equation

In their work LS76 use a simplified cloud scale momentum equation. They assume the cloud-scale mass-flux M_c to be constant over height, reducing eq. 3.8 to:

$$\boldsymbol{R}_c = -\frac{M_c}{\rho} \partial_z (\overline{\boldsymbol{u}} - \boldsymbol{u}_c)$$

To solve the colud-scale horizontal momentum equation $\boldsymbol{u}_c$ at cloud bottom is taken to be equal to the large-scale horizontal momentum $\overline{u}$. In their argumentation entrainment and detrainment are considered conservative processes i.e. there are no apparent sources of momentum or mass in the process of momentum exchange between environment and cloud. Furthermore they assume that, in case cloud velocities are sufficiently large, drag forces will have insufficient time to significantly modify the in-cloud momentum $\boldsymbol{u}_c$. Following this argument, cloud-scale horizontal momentum will therefore approximately be conserved during the ascent. By definition, they therefore neglect the effect of cloud-scale pressure gradients and any entrainment or detrainment of momentum during the transport within the convective column. Their study however was no exhaustive study on the effects of CMT, but rather a proposal for a parametrisation. Later studies intensively worked on the correct incorporation of those effects.

3.4.1. The general approach

The initial assumption is that the cloud-scale mean horizontal momentum equation can be expressed similar to the large-scale mean horizontal momentum-equation. Using the inelastic approximation the equation can be written as:

$$\frac{\rho \partial \boldsymbol{u}_c}{\partial t} + \rho \nabla \cdot (\boldsymbol{u}_c \boldsymbol{u}_c + \overline{\boldsymbol{u}' \boldsymbol{u}'}_c) + \frac{\partial}{\rho \partial z}(w \boldsymbol{u}_c + \overline{w' \boldsymbol{u}'}) + \rho f \boldsymbol{k} \times \boldsymbol{u}_c = -\nabla p_c$$

ρ is the air density, f is the Coriolis parameter, ∇ represents the horizontal gradient operator. Here it is assumed that the eddy momentum fluxes on the cloud-scale are negligible. They influence the very small scales which shall be omitted in the grand scheme. Thus, we are left with:

$$\frac{\partial \boldsymbol{u}_c}{\partial t} + \nabla \cdot (\boldsymbol{u}_c \boldsymbol{u}_c) + \frac{\partial}{\partial z}(w_c \boldsymbol{u}_c) + f \boldsymbol{k} \times \boldsymbol{u}_c = -\frac{1}{\rho} \nabla p_c$$

Following Gregory, Kershaw, and Inness [10][12], it is assumed that the convection averaged over the domain is steady, so time derivatives are zero. The last term on the LHS is the Coriolis-Force. This term is assumed to be negligible, since it's effect on the cloud-scale momentum is rather small, due to the comparatively small area of the convective columns.
In consequence expressions are needed for the horizontal advection of horizontal momentum, the vertical advection of horizontal momentum and the cloud-scale pressure gradient:

$$\nabla \cdot (\boldsymbol{u}_c \boldsymbol{u}_c) + \frac{\partial}{\partial z}(w_c \boldsymbol{u}_c) = -\frac{1}{\rho} \nabla p_c$$

The easiest to find is the vertical advection- term. Using 3.11 it may be expressed:

$$w_c \frac{\partial}{\partial z}(\boldsymbol{u}_c) = \frac{1}{\rho} vg \frac{\partial}{\partial z}(M_c \boldsymbol{u}_c)$$

3.4.2. The horizontal-advection-term

Shapiro and Stevens [20] applied the concept of entrainment and detrainment rates to the conservation of cloud momentum:

$$\frac{\partial M_{ci}}{\partial z} = \epsilon_i - \delta_i \tag{3.21}$$

Where ϵ_i and δ_i are the rates of entrainment and detrainment of cloud-mass. Using this 3.8 may be expressed as:

$$\boldsymbol{R}_c = (\overline{\boldsymbol{u}} - \boldsymbol{u}_c) \cdot \epsilon_i - (\overline{\boldsymbol{u}} - \boldsymbol{u}_c) \cdot \delta_i) + M_c \frac{\partial}{\partial z} (\overline{\boldsymbol{u}} - \boldsymbol{u}_c)$$

The first two terms denote the entrainment and the detrainment of momentum respectively. Consequently the vertical divergence of the cloud-scale mass-flux acts to exchange cloud momentum with the environment. Contrary to SL76, where CMT acts like an elevator only affecting the levels at the bottom and the top of the convective column, in this model CMT affects the environment surrounding the convective column. Usually it is assumed that the entrained air has the environmental momentum and the detrained air the cloud-scale momentum [20][22][10][12], therefore:

$$\nabla \cdot (\rho \boldsymbol{u}_c \boldsymbol{u}_c) = \overline{\boldsymbol{u}} \cdot \epsilon_i - \boldsymbol{u}_c \cdot \delta_i$$

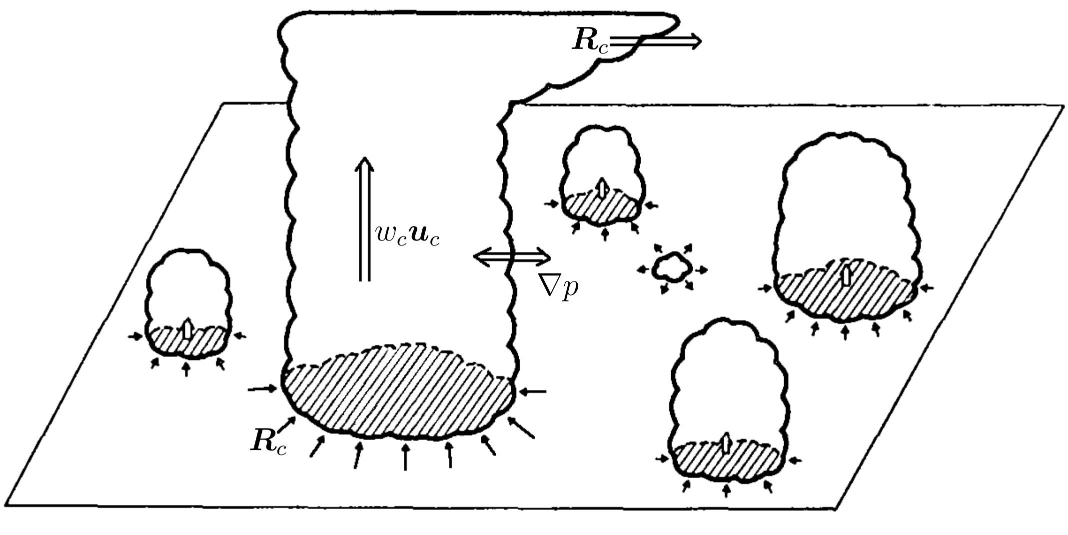


Figure 3.3.: Momentum is entrained and detrained at different levels of the convective column due to cumulus friction. Horizontal momentum is carried upwards by vertical advection being acted upon by the pressure gradient in the process.

A height dependent equation for the cloud-scale vertical mass-flux has been presented (eq. 3.20), the concept of entrainment and detrainment rates is therefore not needed. Entrainment and detrainment appear wherever the gradient of the cloud-scale vertical mass-flux is positive or negative, respectively.
However using the concept the horizontal advection of cloud-scale momentum is expressed here by the exchange of momentum between environment and cloud. This exchange is expressed using the cumulus friction parametrisation 3.8.

$$\nabla \cdot (\boldsymbol{u}_c \boldsymbol{u}_c) = \boldsymbol{R}_c$$

Were all other terms neglected, the momentum advected vertically inside the cloud would therefore be balanced by the momentum horizontally advected across the boundaries of the cloud.

3.4.3. The pressure-perturbation-term

In the years after the publication of the paper by Lindzen and Schneider several studies on the pressure gradients or preturbation pressure fields within convective cells have been published. According to studies by LeMone [14] [13] on lines of cumulonimbus during GATE in September 1974, the vertical transport of horizontal momentum is connected to the vertical wind shear. In squall lines the momentum transport seems to be upgradient normal to a line of cumulonimbus and downgradient parallel to it, while when the convection is less organised, i.e. when convective columns are more randomly distributed and smaller in size both components of horizontal momentum are transported downgradient. These results show the importance of these gradients on the momentum transported and thus several authors worked on new parametrisations extending the CMT[6][27][28][10][12][20][22].

The idea behind formulations incorporating the cloud-scale pressure gradients is that it acts on the cloud-scale horizontal momentum effectively decreasing the difference between cloud and environmental momentum. The problem was most rigorously approached in a study by Cho [6] and later again by Zhang and Cho [27][28](ZC95). They found the effect of the pressure term to be dependent on cloud form and internal structure. The formulation used by most studies today is based on a study by Wu and

Yanai [22] who found the relation:

$$
\begin{aligned}
-\frac{1}{\rho}(\frac{\partial p'}{\partial x})_{ci} &= \gamma_{ki}\frac{\partial \overline{u}}{\partial z}w_{ci} \\
-\frac{1}{\rho}(\frac{\partial p'}{\partial xy)_{ci}} &= \gamma_{li}\frac{\partial \overline{v}}{\partial z}w_{ci} \\
\gamma_{ki} &= \frac{2k_i^2}{(k_i^2+l_i^2+m^2)} \\
\gamma_{li} &= \frac{2l_i^2}{(k_i^2+l_i^2+m^2)}
\end{aligned}
$$

k,l and m spectrally characterise the updraft in x,y and z, respectively. The use of k and l makes it possible to include organised convection, i.e. squall lines ($l^2 \gg k^2$) or disorganised convection ($k \approx l$). In the later case $0 < \gamma_k \approx \gamma_l y < 1$, and therefore $\boldsymbol{u}_c$ will never exceed the environmental wind in magnitude. In contrast in the former case, since $\gamma_l \approx 0$ and $0 < \gamma_k < 2$, the line-normal component and the line-parallel component will behave quite differently. This mirrors what has been found in measurements by LeMone. Usually disorganized convection is assumed. Studies by Gregory et. al. [10] [12] use a value of $\gamma_k = \gamma_l = 0.7$ while a more recent study by Zhang and Wu [26] uses a value of $\gamma_k = \gamma_l = 0.55$. The latter value is used, since results obtained are in agreement with the earlier study by ZC95.
A recent study by Romps [18] proposes to turn the parameter to zero, effectively only using the parametrisation by Schneider and Lindzen [19]. However the parametrisation above shall be incorporated for a better physical representation. The pressure perturbation term will be incorporated using:

$$
\frac{1}{\rho}\nabla\frac{\partial p_c}{\partial z} = \gamma\frac{M_c}{\rho}\frac{\partial \overline{\boldsymbol{u}}}{\partial z}
$$

With $\gamma = 0.55$ according to Zhang and Wu.
Note that this parametrisation with a fixed γ does not represent the effects of organized convection. The up-gradient transport found by LeMone [14] [13] may not be represented in this study. Models aiming to include these effects need a higher spacial resolution than the KMCM. Furthermore a model of the cloud distribution is needed.

3.5. The complete set of equations

Combining the findings from sections 3.2,3.3 and 3.4 the model equations for the parametrisation are:

$$\boldsymbol{R}_c = -\frac{1}{\rho}\partial_z(M_c(\overline{\boldsymbol{u}} - \overline{\boldsymbol{u}}^c)) \tag{3.22}$$

$$M_c(z) = \frac{\dot{Q}_c\rho}{g} \tag{3.23}$$

$$\frac{\partial}{\partial z}(M_c\boldsymbol{u}_c) = \partial_z(M_c(\overline{\boldsymbol{u}} - \boldsymbol{u}_c)) - \gamma M_c\frac{\partial\overline{\boldsymbol{u}}}{\partial z} \tag{3.24}$$

Expressions for the cloud-scale horizontal momentum (eq. 3.24) and vertical mass-flux (eq. 3.23) in respect to the large scale quantities have been found. The equation (eq. 3.22) yields the large scale change of horizontal momentum due to CMT.

Chapter 4.

Application to the KMCM

4.1. Implementation

The KMCM model is a General Circulation Model (GCM) which can be adapted to different vertical resolutions. Common resolutions for long-term modelling are T32L70, with 450 km horizontal resolution and 70 layers up to a height of 130 km, and T42L115, with 300 km horizontal resolution and 115 layers up to a height of 125 km. It is calculated using a spectral-transformation for vorticity, enthalpy, surface pressure, surface temperature and tracers. The model has parametrisations for gravity waves, radiation and moisture cycles, including convection.
The output of the model is given as a series of spherical harmonics and post-processing is needed to evaluate the computed data. This allows to test new parametrisations 'offline', i.e. using outputs and computing the next time step including the new parts to see whether they work properly or not.

The model makes use of so called model-levels (l) and hybrid-levels ($l \pm \frac{1}{2}$). Quantities used for derivatives are computed on the hybrid levels. This is to ensure correct gradient values on the model-layers. Outputs are generated on the model layers.

Cloud-scale mass-flux

The discretisation of the mass-flux equation expressed thus:

$$M_c(l+\tfrac{1}{2}) = \frac{\dot{Q}_c(l+\tfrac{1}{2}\cdot\rho(l+\tfrac{1}{2})}{g} \tag{4.1}$$

$$\frac{\partial M_c}{\partial z}(l) = \frac{M_c(l-\frac{1}{2}) - M_c(l+\frac{1}{2})}{\Delta z} \tag{4.2}$$

Where Δz is derived from the hydrostatic pressure equation:

$$\Delta z = \frac{g\cdot\rho}{\Delta p} \tag{4.3}$$

At the levels $-\frac{1}{2}$ and $l_{max}+\frac{1}{2}$ the mass-flux is set to zero. Everywhere else it should follow the behaviour of the convective heating $\dot{Q}_c$.

Cloud-scale momentum

In order to discretise the momentum equation, $M_c(l)$ is expressed as the average of its two neighbouring hybrid level values to get:

$$\begin{aligned}\boldsymbol{u}_c(l-\tfrac{1}{2}) = (&\boldsymbol{u}_c(l+\tfrac{1}{2})\cdot M_c(l+\tfrac{1}{2})\\ &+\frac{1}{2}\overline{\boldsymbol{u}}(l-\tfrac{1}{2})\cdot((1-\frac{\gamma}{2})\cdot M_c(l-\tfrac{1}{2}) - \frac{\gamma}{2}\cdot M_c(l+\tfrac{1}{2}))\\ &-\frac{1}{2}\overline{\boldsymbol{u}}(l+\tfrac{1}{2})\cdot((1-\frac{\gamma}{2})\cdot M_c(l+\tfrac{1}{2}) - \frac{\gamma}{2}\cdot M_c(l-\tfrac{1}{2})))/M_c(l-\tfrac{1}{2})\end{aligned}$$

Here it is implied that $M_c(l-\frac{1}{2}) \neq 0$. Furthermore $\boldsymbol{u}_c$ is initialised to be $\boldsymbol{u}_c = \overline{\boldsymbol{u}}$ along the whole column, guaranteeing zero cumulus momentum transport when there is no convection, where $\gamma = 0.55$ and the zonal and meridional components are computed individually.

Cumulus friction

The momentum-flux through cumulus convection is defined as:

$$\boldsymbol{F}_c = M_c\cdot(\overline{\boldsymbol{u}} - \boldsymbol{u}_c)$$

Utilising this definition of the flux, the initial values are set to zero for two reasons: to guarantee the boundary conditions for the surface and the top of the atmosphere, where $\boldsymbol{F}_c = 0$, and to guarantee zero transport where no convection occurs. Finally $\boldsymbol{R}_c$ is computed:

$$\boldsymbol{R}_c(l) = -\frac{1}{\rho(l)} \frac{\boldsymbol{F}_c(l - \frac{1}{2}) - \boldsymbol{F}_c(1 + \frac{1}{2})}{\Delta z(l)}$$

Note that $\boldsymbol{R}_c$ is computed on model layers.

Output

The output of other quantities is generated by taking the average of two values on hybrid layers above and below the layer under examination, i.e.:

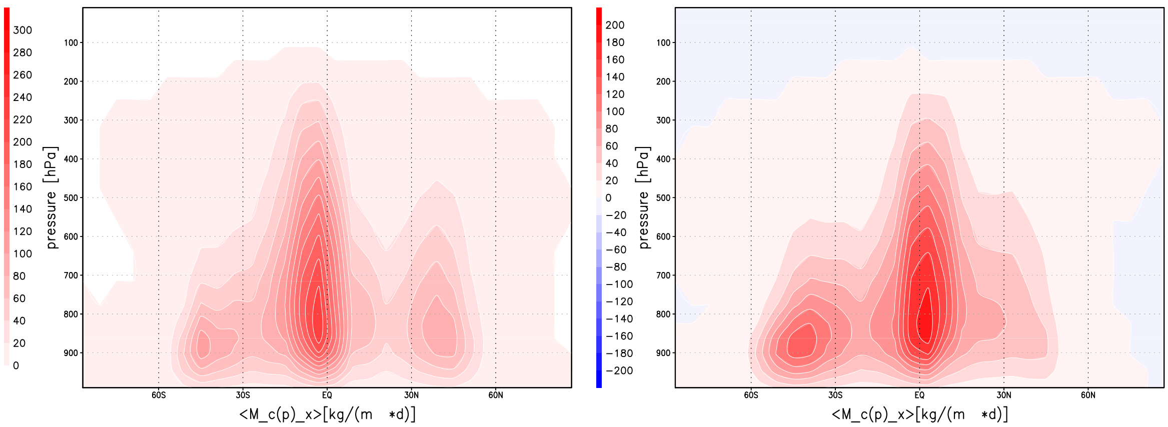
$$I(l) = (I(l - \tfrac{1}{2}) + I(l + \tfrac{1}{2})) \cdot \frac{1}{2} \tag{4.4}$$

4.2. Results

The parametrisation was tested for a series of 'offline' snapshots of the KMCM. The Cumulus-Friction R_c, cloud-scale mass-flux M_c and cloud-scale horizontal velocity $\boldsymbol{u}_c$ have been computed for the model year January to December 1997. A sample was taken every five days, averaged over ten time steps of the KMCM. All information taken from the computations are zonal averages. The results presented here are averaged over the winter months December, January and February (DJF) and the summer months June, July and August (JJA). The CMT model uses the convective heating rates Q_c from the convection routine of the KMCM. As mentioned earlier, the approximations used for the designation of the reference temperature profiles are only correct to a certain height. To account for that the convective routine in the KMCM has a lid-condition. This is applied to the CMT scheme in order to keep both routines in a synchronous state. Other inputs from the KMCM are the horizontal wind profiles as well as the vertical pressure and density profiles. The Cumulus-Friction $\boldsymbol{R}_c$, cloud-scale mass-flux M_c, cloud scale velocity $\boldsymbol{u}_c$ and cumulus induced vertical momentum transport $\boldsymbol{F}_c$ are given as outputs of the routine.

4.2.1. Cloud-scale vertical mass-flux

Figure 4.1 shows the averages of the vertical cloud-scale mass-flux. During DJF a structure of three distinct areas of mass-fluxes between $60°$ and $30°$ S, at the equator and between $30°$ and $60°$ N can be seen. During the summer the maximum between $30°$ and $60°$ N has vanished, while the maximum at $60°$ and $30°$ S is somewhat more pronounced. Its elongated maxima reach values of $200\ \frac{kg}{m^2 d}$ and stretch up to 200 hPa at the equator, while at the sides the maxima reach values of only $120\ \frac{kg}{m^2 d}$ stretch up to 800 hPa.



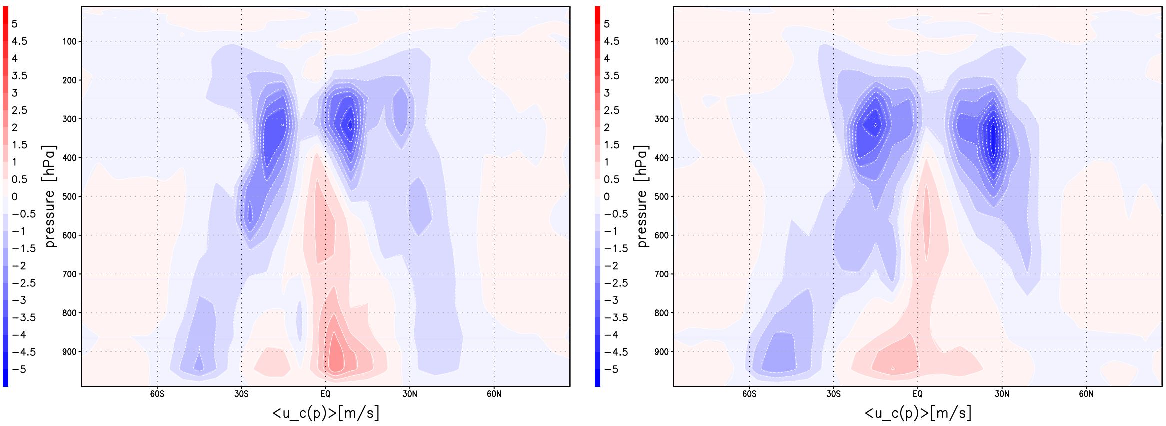
(a) M_c in $\frac{Kg}{m^2 d}$ during DJF **(b)** M_c in $\frac{Kg}{m^2 d}$ during JJA

Figure 4.1.: Averages of the vertical cloud-scale mass-flux during the summer and winter months

4.2.2. Cloud-scale horizontal-velocities

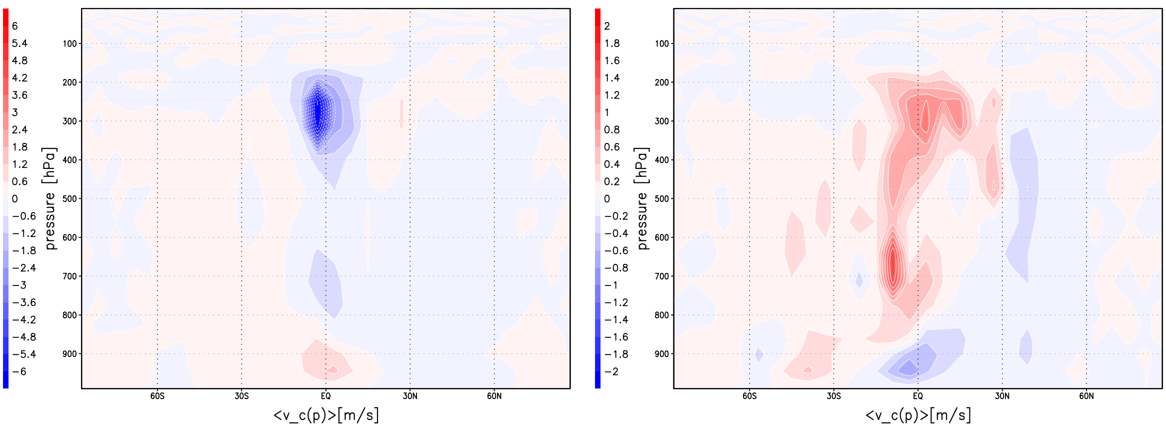
The difference between large-scale average horizontal wind velocity and cloud-scale horizontal wind velocity is plotted in figure 4.2 and figure 4.3. The structure of the zonal wind profile is largely similar for winter (DJF) and summer (JJA). At the equator and up to the height of 400 hPa cloud-scale momentum is larger than the environmental momentum. Above and towards the poles cloud-scale momentum is lesser than the environmental momentum, indicating friction.

The meridional cloud-scale momentum during DJF (fig. 4.3a) has several pronounced maxima at the equator. The most pronounced lies at 350 hPa indicating strong friction. During the summer (JJA) the structure is mirrored along the equator, with cloud velocities below the environmental wind speeds close to the surface and on the northern hemisphere and large areas above the equator between 800 hPa and 200 hPa where the cloud-scale velocities are up to 2 $\frac{m}{s}$ larger than environmental velocities.



(a) $u_c - \overline{u}$ in $\frac{m}{s}$ during DJF **(b)** $u_c - \overline{u}$ in $\frac{m}{s}$ during JJA

Figure 4.2.: Difference between environmental and cloud-scale horizontal velocities



(a) $v_c - \overline{v}$ in $\frac{m}{s}$ during DJF **(b)** $v_c - \overline{v}$ in $\frac{m}{s}$ during JJA

Figure 4.3.: Difference between environmental and cloud-scale horizontal velocities

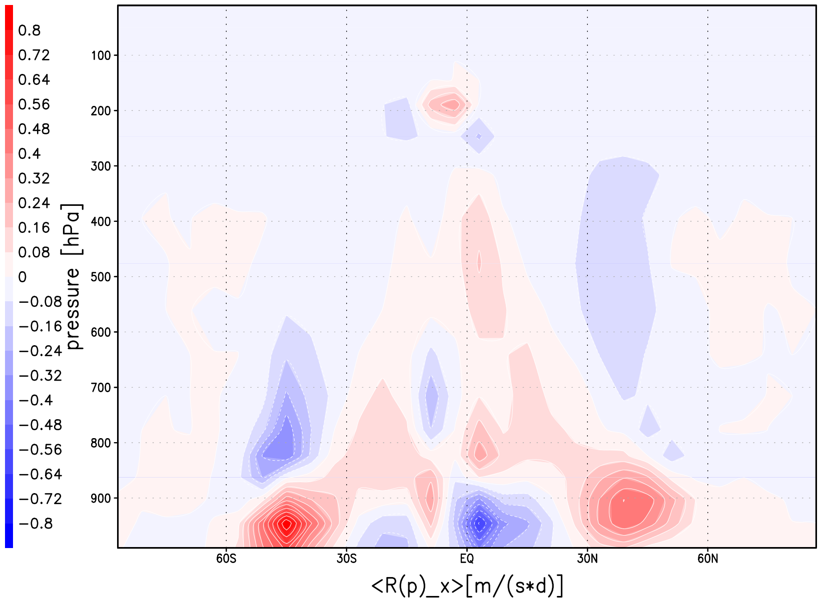


Figure 4.4.: Meridional averages of the zonal component of $\boldsymbol{R}_c$ during the winter months.

4.2.3. Cumulus Friction

Winter (DJF)

Figure 4.4 shows the zonal component of the cumulus friction resulting from the simulation averaged over DJF. The upward extent of the convective momentum transport is largely dependent on the convective mass-flux, hence it is only in the tropics, where deep convection occurs, that the effect reaches into the upper troposphere. Westerly cumulus friction reaches maxima of 0.8 $\frac{m}{s \cdot d}$. Easterly cumulus friction goes up to values of -0.6 $\frac{m}{s \cdot d}$. The northerly (southerly) cumulus friction reaches 0.4 $\frac{m}{s \cdot d}$ (-0.3 $\frac{m}{s \cdot d}$). There is a distinct structure with westerly friction between 900 hPa and 700 hPa in the range between $0°$ and $30°$ and reaching down below 900 hPa towards the poles between $30°$ and $60°$ on both hemispheres. These two areas of westerly friction are separated by two areas of easterly friction located close to the equator, one below on the northern side being the maximum easterly friction and the other at 700 hPa. Another distinct area of westerly friction occurs at 200 hPa at the equator, accompanied by two peaks of easterly friction.

The meridional component of the cumulus friction during DJF shown in figure 4.5 behaves significantly different from its zonal component. Its northerly maximum lies between 900 hPa and 800 hPa at a value of 0.6 $\frac{m}{s \cdot d}$. Its southerly maximum is located

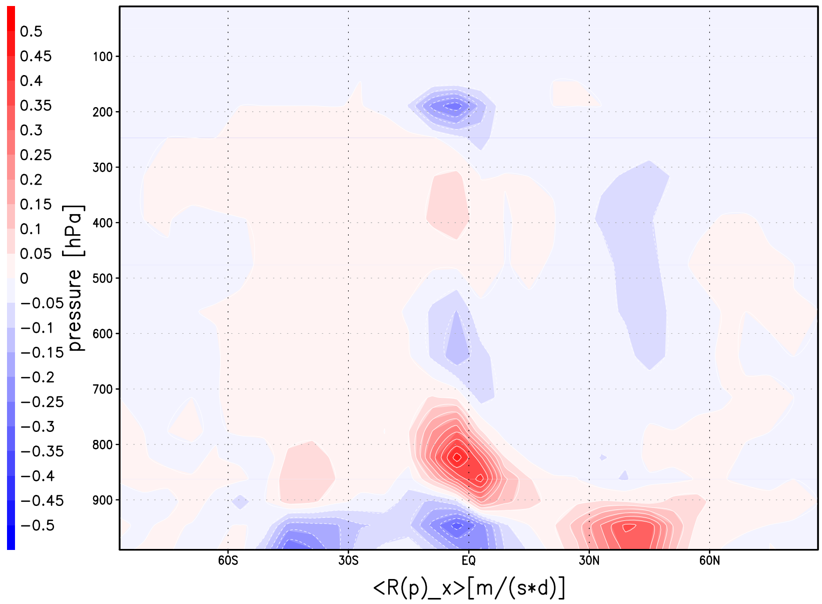


Figure 4.5.: Meridional averages of the meridional component of $\boldsymbol{R}_c$ during the winter months (DJF).

at around 200 hPa reaching -0.4 $\frac{m}{s \cdot d}$. The vertical structure of the cumulus friction is similar, reaching up to 200 hPa at the equator and being limited to 800 hPa between 30° and 60° on both hemispheres.

Summer (JJA)

A second evaluation has been made for the summer. Generally the effect for both the zonal and meridional component of the cumulus friction are concentrated on the winter hemisphere, and nonapparent on the summer hemisphere. Figure 4.6 shows the zonal component. While the vertical extent is comparable, the overall structure is distinctly different from the winter months. The maxima lie on the winter hemisphere. Westerly friction with a maximum value of 1 $\frac{m}{s \cdot d}$ is found between 60° and 30° S and below 900 hPa, above it at 800 hPa the maximum easterly friction is found reaching values of -0.6 $\frac{m}{s \cdot d}$.

Similar results are found for the meridional component (fig. 4.7). Its maxima again lie between 60° and 30° S with maximum values of -0.45 $\frac{m}{s \cdot d}$ southerly friction close to the surface and 0.2 $\frac{m}{s \cdot d}$ northerly friction located above between 900 hPa and 800 hPa.

Above 800 hPa influences on the meridional momentum are negligible. The distinct maxima of friction found above 700 hPa during DJF are not found during JJA.

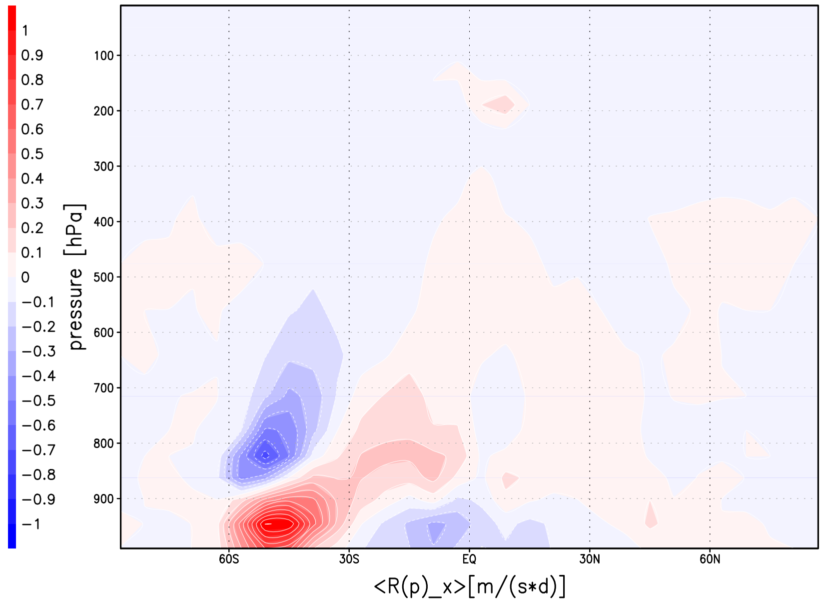


Figure 4.6.: Meridional averages of the zonal component of $\boldsymbol{R}_c$ during the summer months (JJA).

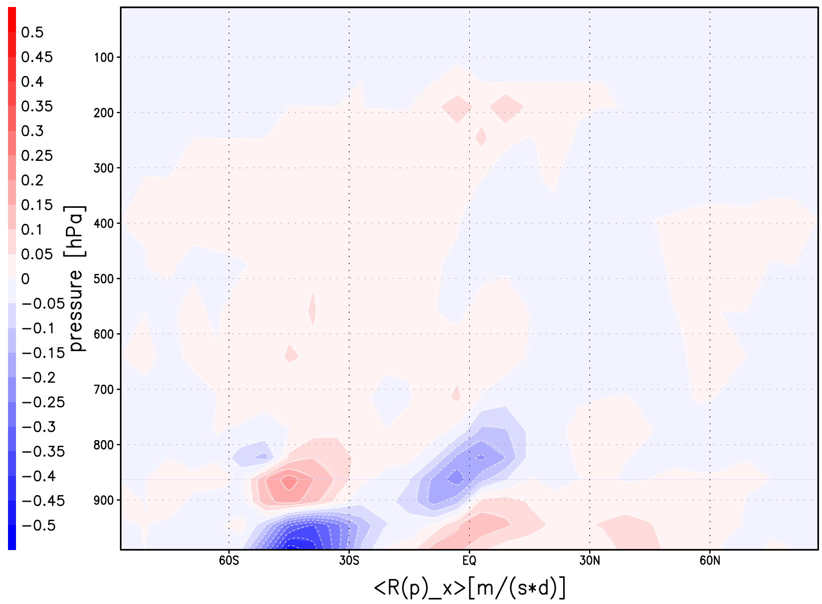
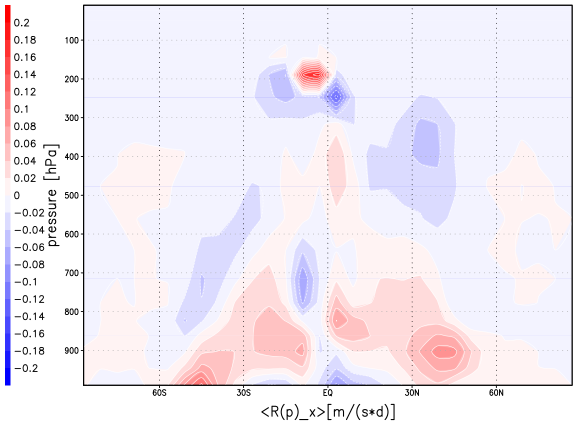


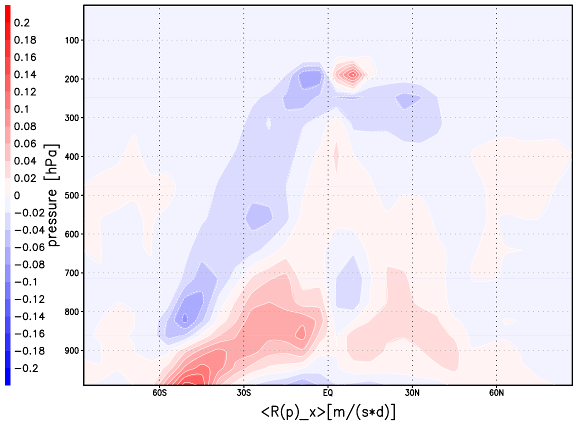
Figure 4.7.: Meridional averages of the meridional component of $\boldsymbol{R}_c$ during the summer months (JJA).

4.2.4. Impact of pressure gradient

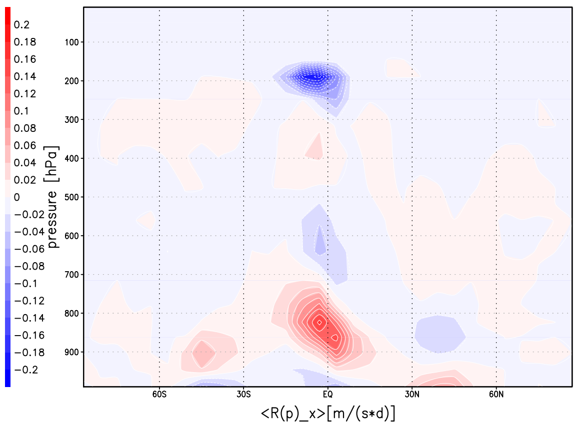
To quantifiy the effects of the parametrisation of the pressure gradient proposed by Gregory et. al [10][12] and Wu [21], a test run was made without the pressure gradient. The results from this run were subtracted from the results obtained with the gradient parametrisation. Results (fig. 4.8) show an increase of the overall tendencies of the friction throughout the whole atmosphere by $\pm\, 0.2\ \frac{m}{s \cdot d}$. This amounts to an increase of approximately 20% in friction through cumulus momentum transport by introducing the pressure gradient term.



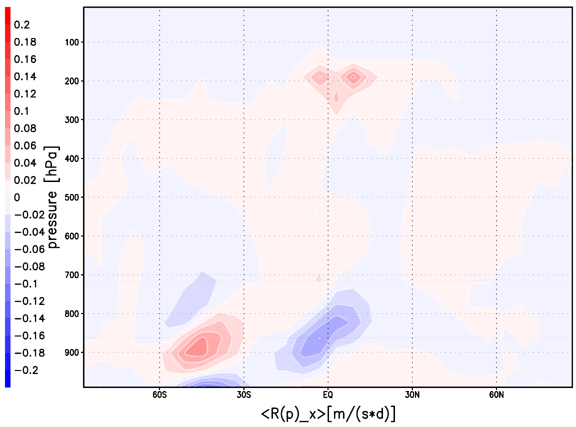
(a) Isolated effect of the pressure-gradient during DJF.



(b) Isolated effect of the pressure-gradient during JJA.



(c) Isolated effect of the pressure-gradient during DJF.



(d) Isolated effect of the pressure-gradient during JJA.

Figure 4.8.: Effects of the pressure gradient term on cumulus friction.

4.2.5. Evaluation

In their paper Richter and Rasch [17] compare the two schemes presented by Schneider and Lindzen and Gregory et. al.. They present computations made for DJF (their figure 1) using the Community Atmospheric Model, Version 3 (CAM3). In comparison the overall structure for the zonal component of the cumulus friction is quite similar to the findings presented here. The meridional component of the cumulus friction has a similar distribution throughout the atmosphere, but the orientation of the friction is different from results found here. Below 800 hPa, in the region between $30°$ S and the equator, Richter and Rasch find a pronounced area of northerly friction, where here southerly friction is found. This may be explained by different wind fields used for the simulation; the year for which their simulation was made is not specified and differences in the models used may account for these differences. For the magnitude of zonal cumulus friction, they find ranges between $\pm$ 3.5 $\frac{m}{s \cdot d}$ without pressure gradient and 0.8 $\frac{m}{s \cdot d}$ and -1.4$\frac{m}{s \cdot d}$ with pressure gradient included. Former values are far beyond what is found here, the latter are well in agreement with what is found using the parametrisation presented here. The same can be said for the meridional components of the cumulus friction.
The influence of the pressure gradient term was found to be of rather small impact on the cumulus friction. Additionally, rather than decreasing the cumulus friction it increases it, which is contrary to the findings presented by Richter [17] and what is expected. However according to what is found in a study by Romps [18], the effects of the pressure gradient are negligible. He concludes the parametrisation made for the pressure gradient, though it represents the cloud-scale physics more realistically, to be of little effect and proposes to leave it out of cumulus momentum transport parametrisations. His findings are confirmed through the results found in this study.

Chapter 5.

Conclusion and Outlook

A convective momentum transport (CMT) scheme was developed and implemented in the Kühlungsborn Mechanistic Climate Model (KMCM). The scheme is based on the scheme proposed by Schneider and Lindzen and was improved following publications by later authors. The scheme consists of a cloud-scale horizontal momentum equation and a cloud-scale mass-flux equation. The results of these are then combined in the parametrisation of the turbulent vertical eddy momentum flux found by Schneider and Lindzen to equate the cumulus friction.

It was found that the parametrisation of convection applied in the respective model has a large impact on the parametrisation of CMT. Adjustments were made to translate the parametrisations from the mass-flux convection schemes used by previous authors into the relaxation kind convection scheme implemented in the KMCM. To that length a cloud-scale horizontal momentum equation and a cloud-scale mass-flux equation were derived. In the cloud-scale horizontal momentum equation the horizontal advection term is set to be equal to the momentum transported over the clouds boundary i.e. the friction between cloud and environment is used to model entrainment and detrainment of momentum.

The clod-scale vertical mass-flux is found using the heating rate specified by the convection scheme of the KMCM. The formulation is found through the application of two different approaches to the problem of representing convection. Following considerations of distinct parcels to define the convective available energy. It is understood that energy is accumulated before a convective event and translated into kinetic energy in form of a vertical mass-flux and eventually dissipated during the convective event. Based on this the heating rates are considered the result of work done on the system, allowing to 're engineer' the cloud-scale mass flux.

Results from 'offline' simulations with the KMCM are presented and evaluated. It is found that spatial distribution as well as magnitude are in agreement with findings by Richter and Rasch [17]. The overall impact of the much discussed pressure gradient could however not be reproduced. This can be explained by the insignificance or insufficiency of the applied parametrisation of the pressure gradient, following the arguments of Romps [18]. On the other hand it is also possible that the parametrisation of the cloud-scale horizontal advection term is responsible. This was modified, in respect to other schemes, to adapt to the different representation of cloud-scale mass-fluxes in this CMT scheme. The role of pressure gradients and the resulting changes in the direction of cumulus momentum transport are a wide field that could only be superficially be touched in this work. Based on more recent studies however it is evident that a correct representation of upgradient and downgradient momentum transports due to cumulus convection can not be fully represented by a relaxation-type convection scheme. The pressure gradient parametrisation used here is an approximation. Cloud Resolving Models (CRM's) are used to model orientation and organization of convection. Without such a distinct representation of clouds it is difficult to account for such effects. However it is the authors opinion that the gradient term should be both included as well as improved in the future, since there is evidence of the importance of its effect on CMT.

The vertical momentum is in this simulation implemented as a catalyst for the effect of cumulus transport of horizontal momentum. It is assumed that it can be represented as a function of the heating rate and the acceleration of the vertical mass-flux, which is approximated to be g. This representation yields adequate results, however to guarantee physical representability the idea of a cloud-scale vertical velocity equation should be reviewed. This representation could yield interactions between clouds and environment, resulting in vertical acceleration of the environment and generally in a more realistic representation of the cumulus friction and vertical transports of tracers and mass through convection. Furthermore the effect of accelerated air moving past the LNB, referred to as overshoot that is not simulated here, could be included using this vertical momentum equation. It is suggested that future considerations should include these effects to ensure little errors are made in specifying the cloud-scale mass-flux, since uncertainties in the determination of it result in uncertainties in the CMT.

In the future 'online' simulations with the KMCM will be made. Using the results from these simulations the parametrisation presented here can be further evaluated. The results achieved by 'online' runs of the KMCM may be compared to the results of previous authors. Based on previous publications general improvements in the representation of the atmospheric circulation by the model are expected.

Appendix A.

Comotra.f

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